

# Linear Program Modeling for Determining the Value of Irrigation Water

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## ABSTRACT

The main objective of this paper was to estimate the marginal value of irrigation water through linear programming technique. Linear programming models for three model farm sizes (small, medium, large) were developed to estimate the short run marginal value of water and predict the impact of different water supply scenarios on cropping pattern, cropping intensity and net income of the representative farms. The results showed that shadow price of water varied over time (by months). The value of water ranged from Rs. 0.45 to Rs. 1.31 in small farm, Rs. 0.86 to Rs. 1.64 in medium farm and Rs. 0.95 to 1.84 in large farm, respectively. Results revealed that scarcity of water adversely affects the cropping intensity and net income of representative farms.

**Key Words:** Water value; Linear programming; Cropping intensity

## INTRODUCTION

The water crisis in Pakistan is quite severe and long-term forecast of the experts is even bleaker. Meteorologists, blame the prolonged drought because La Nina weather phenomenon have warned that the country has entered into a dry cycle and could expect drought-like conditions to return after every six years (Ohlsson, 2001). Experts predict that with the prevailing consumption and population growth rates, one out of three people in Pakistan will face shortages of water. The drought has highlighted the lack of definitive water policy by successive governments and exposed serious shortcomings in Pakistan's water storage and antiquated irrigation systems (Ohlsson, 2001).

Water pricing policy has the potential to mitigate water scarcity. Because of its key role in managing water demand and augmenting water supply, water pricing is an important policy instrument for creating incentives to conserve and allocate water efficiently. By providing financial justification for developing additional supplies from conventional and non-conventional sources, pricing policy can make more water available to users. The water pricing policies being pursued in most countries including Pakistan fail to perform the vital roles due to faulty approaches and inappropriate institutions, both of which have their roots in political economy (Saleth, 2001).

Linear programming (LP) operates with the objective of maximizing economic returns subject to resource, production and policy constraints and is able to adapt to the supply conditions (i.e. drought conditions). This model is unique in its ability to identify specific monthly water allocations that exactly match the State's supply to demand by assessing the willingness to pay off different agricultural water users for a reliable water supply. The willingness to pay for a specific quantity of water can be inferred from the imputed shadow value on the water to the users.

This approach explicitly recognizes the effect of high user prices on water demand and conversely the effect of the

willingness to pay for water reliability. Although the model is constrained by spatial water, which includes physical limitations on annual water availability, the optimal solution allows for transfer of water between different months such that the marginal value of water by month and crop is equated. A shadow value represents the "true" value of an additional unit of water to a buyer in the region. Generally speaking, this additional unit of water would in turn produce additional agricultural output, whose value is dependent upon the type of crop grown and the price that is specific to a region.

The objective of LP is to maximize net returns from agricultural production, subject to production and resource constraints on water and land. Production constraints are in the form of functional relationships describing the productive tradeoffs between land and water use efficiency, in conjunction with capital cost expenditures. The model distributes water supply based upon annual water allocation, water costs and the production opportunities facing the region. The model assumes a perfectly competitive market structure such that producers are not able to influence prices in either input or output markets. The main objective of the paper was to construct the LP model for representative agriculture model farms for estimating (short run) marginal value of irrigation water or shadow price of water on annual and monthly basis. The sensitivity analysis was also done for different cropping pattern and cropping intensity.

## MATERIALS AND METHODS

**Linear programming (LP) method.** An LP problem contains several essential elements. First there are decision variables ( $x_i$ ) the level of which denotes the undertaken of the respective un-knowns,  $k$  ( $i = 1, 2, \dots, k$ ). Second is the linear objective function, where the total objective value

$$Z = p_1x_1 + p_2x_2 + \dots + p_nx_n.$$

Where  $p_i$  is the contribution of each unit of  $x_i$  to the objective function. The problem is subject to constraints of

m. An algebraic expression for the *i*th constraint is  $w_{j1}x_1 + w_{j2}x_2 + \dots + w_{jn}x_n \leq v_j$  ( $j = 1, 2, \dots, m$ ) where  $v_j$  denotes the upper limit or right hand side imposed by the constraint and  $w_{ij}$  is the use of the items in the *i*th constraint by one unit of  $x_i$ . The  $p_j$ ,  $v_j$  and  $w_{ij}$  are the data (exogenous parameters) of the LP model" (Bruce, 2000).

**Assumptions of LP.** Linear programming is a widely used technique for determining the optimal allocation of scarce resources, although its applicability is subject to certain underlying assumptions. Beneke and Winterbaer (1973) summarize the basic assumptions underlying the application of linear programming to a problem as:

**Linearity and additive:** all activities use inputs in a fixed proportion and exhibit constant returns to scale. This means that the activities have additive impacts on the objective function and on resource use.

**Divisibility:** means the potential breakdown of the inputs into any number or fraction of a unit.

**Finiteness:** means we have a limited number of inputs and each with a certain limit to produce certain enterprises.

**Single value expectations:** there is a single value for the objective function coefficients (maximization or minimization problem).

Two key assumptions of LP models when used to impute the value of resources are: profit maximization (market clearing conditions) and perfect foresight of the market conditions. Under real world situation it is difficult to fulfill these assumptions, especially in traditional economies. LP models are commonly formulated in the form of 'm x n' matrix where the 'm' rows comprise the constraint inequalities and the 'n' columns represent the various activities under consideration. Usual LP model can therefore be described by four components, which are used to create, the 'm x n' matrix: the activities, constraint values, technical coefficients and objective function (Elobeid, 1982).

The LP technique may be appropriate in an aggregate application if adequate empirical data are available, because it furnishes a convenient format for stating problems of maximization or minimization with its solution through computer routines (Reuss, 1980). Each optimal linear programming solution includes a set of 'shadow prices', which tell the analyst about constraints; most strongly affect the value of the optimal solution (Elobeid, 1982).

**Structural specification of model and source of data.** The main objective was to determine the optimal allocation of resources among competing products or activities. Every maximization (minimization) problem in linear programming has a corresponding minimization (maximization) problem. The original problem is called the primal and the corresponding one is dual. The power of LP is that it provides simultaneously optimal solutions for the various activities (in the primal problem) and shadow prices of all commodities and resources (in the dual problem). The primal method solves the problem as formulated, e.g. maximizing the value of an objective function subject to

fixed quantities of inputs, while the dual turns the problem around and minimizes the value of each resource constraint subject to the values of the activities comprising the primal function.

**Study area.** The data for LP model was obtained from Lalian distributory in Sargodha district. The representative model farms of 6 acre (small farm), 12 acres (medium) and 25 acres (large farm) farms were taken. The farm budgets developed into LP matrix with appropriate coefficients and constraints. The data included crop yields, all farm inputs, water, labor etc. specified in the model.

**Activities and constraints.** The model has two types of activities: field crop activities and resource augmenting activities. Seven crop activities were included: wheat, rice, sugarcane, cotton, sorghum, maize and Rabi fodder. Regarding constraints, land and water constraints for each size of farm were applied. For small, medium and large farm sizes, land was constrained by 3, 6 and 7 acres, respectively. Water was constrained according to its availability on monthly basis. A crop calendar was developed where water was given on monthly basis in the LP matrix. Labor and water limits were also defined.

**The technical coefficients matrix.** The matrix of technical coefficients represented the quantity of resources or inputs needed for production of the mix of crop activities during a year (i.e. monthly water & labor requirements).

**The objective function.** The objective function of model was to maximize the net return to water, land and labor of a tenancy at Lalian Distributory in Sargodha District, subject to certain technical coefficients of production function and certain level of resource availability.

**Description of alternative models.** For the sensitivity analysis, the available water was discounted to 11, 20.9, 30.7, 40.4, 50.2 and 60.1%. Six models were estimated on each farm size category. Model 1 served as base model. The main purpose was to determine the allocation of resources with reducing amount of water. First three model of small farm size provided the similar results due to the land constraint. Reduced water supply did not show any impact on net returns and cropping intensity.

**Change in net income method.** In practical applications, irrigation water is often valued with the Change in Net Income (CINI) method. The increment in net income is associated with adding water to a production process as willingness to pay for the incremental water. A process very similar to that used for residual imputation can represent this approach.

A more general multi-product/multi-input production function can be written as:

$$f(Y_1, \dots, Y_m; X_1, \dots, X_n) = 0 \quad (i)$$

Where:

$Y_i$  = No. of feasible crops ( $i = 1 \dots n$ )

$X_j$  = No. of production inputs ( $j = 1 \dots m$ )

The net income (denoted by Z) from producing a given set of crops can be obtained by subtracting the gross income of all crops from the total cost for all inputs. It is

represented by

$$Z = \sum_{i=1}^m (Y_i P_{yi}) - \sum_{j=1}^n (X_j P_{xj}) \quad (ii)$$

The Change in Net Income is

$$\Delta Z = Z_1 - Z_0 \quad (iii)$$

Where,  $Z_0$  and  $Z_1$  refer to 'without project' and 'with project' situation, respectively. The water resource is designated  $X_i$ . Assuming that the increase in crop production following from the added water supplies is not so large as to influence crop prices, the change in net income ( $\Delta Z$ ) associated with a discrete addition to water supply per unit of time. Unit value of water may be obtained by dividing the expression in equation (iii) by incremental quantity of water ( $\Delta X_i$ ) (Nobe & Sampath, 1986).

The CINI approach requires the same assumptions as of the residual imputation method. The change in net income method can be adapted to mathematical programming models (LP model) of farm situations to approximate a functional relationship between net benefits and irrigation water use (Bowen & Young, 1985; Chaudhry & Young, 1989). The programming model of a representative farm situation is formulated to maximize net return to the residual claimant, (the water resource in this case) subject to constraints on water availability. The model is solved for each of a number of increments of water supply and the net return to each increment of water derived from the incremental change in the objective function (Hussain *et al.*, 1995). LP model provides a useful tool to derive CINI for further derivation of the value of water.

## RESULTS AND DISCUSSION

**Linear programming.** The linear programming (LP) model was solved for a range of water supply levels for each of the varying farm sizes, i.e. small farm (6 acres), medium farm (12 acres) and large farm (25 acres). The estimated monthly water supply was increased/decreased from 10 to 60 percent to perform the sensitivity analysis design to ascertain impact

of water supply on cropping and production variables. The most important of these variables for this study were: cropping pattern, cropping intensity and the short run value of water for both monthly and seasonal periods. The results are discussed for each variable in turn.

**Cropping intensity and cropping pattern.** Cropping intensity refers to the proportion of irrigated land cropped in 12 -month period. The LP approach also optimized the land among crops to get maximum returns. The results of sensitivity analysis in water supply constraints for the three farm sizes revealed that the cropping intensity was sensitive to different water supply constraints. It reduced with the discounted water supply.

The cropping pattern was identified under each of the three farm sizes. Wheat, rice, sugar cane and maize (which were less competitive & more profitable crops to farmer) dominated the cropping area relative to cotton, sorghum and Rabi fodder. Table I shows the cropping intensity and cropping pattern of the sample farms.

**Shadow prices of water.** The LP model provided the shadow prices of water in water deficiency months. With monthly water supply constraints, shadow prices reflected the marginal profit of water to the farmer in each month. Table II revealed how shadow price of water varied over time (by months) and by the discounting water supply. There was shortage of irrigation water supply during the months of January, February, May, October and November. The relative scarcity of water during January, February and May, was due to low rainfall. And during the months of October and November it was due to the limited supply in the delivery system as compared to the requirement (wheat crop was sown during these months).

**Change in net income method.** The basic concept for establishing shadow prices is the notion of willingness to pay as the indicator of value. In accordance with the postulates of diminishing marginal productivity or utility,

**Table I. Cropping Intensity and Cropping Pattern of the Sample Farms**

Farms Category	Models	Wheat	Cotton	Rice	Sugarcane	Sorghum	Rabi fodder	Maize	Total cropped area	Cropping intensity
Small	Model 1	-	-	3	3	-	-	-	6	100
	Model 2	-	-	3	3	-	-	-	6	100
	Model 3	-	-	3	3	-	-	-	6	100
	Model 4	2.5	-	2.1	1.2	-	-	-	5.8	96.67
	Model 5	3	-	1.3	-	-	-	0.5	4.8	80
	Model 6	1.5	-	0.6	-	-	-	0.25	2.35	39.17
Medium	Model 1	2.7	-	4.7	4.6	-	-	-	12	100
	Model 2	3.9	-	4.5	3.6	-	-	-	12	100
	Model 3	6	-	4	1.7	-	-	0.3	12	100
	Model 4	6	-	2.6	-	-	-	1	9.6	80
	Model 5	3	-	1.3	-	-	-	0.5	4.8	40
	Model 6	1.5	-	0.6	0.25	-	-	-	2.35	19.58
Large:	Model 1	7	-	5.4	3	-	-	2.3	17.7	70.8
	Model 2	7	-	4.9	2.4	-	-	2.2	16.5	66
	Model 3	7	-	4.1	1.3	-	-	2	14.4	57.6
	Model 4	6	-	2.6	-	-	-	1	9.6	38.4
	Model 5	3	-	1.3	-	-	-	0.5	4.8	19.2
	Model 6	1.5	-	0.25	-	-	-	0.6	2.35	9.4

(Area = acres, Cropping Intensity = Percent)

**Table II. Shadow Prices of Irrigation Water (Rs./M<sup>3</sup>)**

Farm Categories	Models	Water Deficiency Months	Shadow Prices of Water
Small	Model 1, 2 & 3	-	-
		Model 4	Feb. 1.64 May 1.55
	Model 5	Nov.	0.45
		Jan.	3.17
		May	3.62
Medium	Model 1 & 2	Feb.	5.33
		May	2.11
	Model 3	Feb.	6.95
		May	3.91
	Model 4	Nov.	0.45
		Feb.	6.77
		May	4.09
	Model 5 & 6	Nov.	0.45
		Jan.	5.91
		Feb.	6.77
		May	4.09
Large	Model 1, 2 & 3	Oct.	0.22
		Feb.	5.81
		May	2.53
	Model 4, 5 & 6	Nov.	0.45
		Jan.	12.15
		Feb.	5.61
		May	2.66

**Table III. The Marginal Value of Irrigation Water (Rs./M<sup>3</sup>)**

Farm Categories	Models	Irrigation Water (M <sup>3</sup> /acre)	Net Returns (Rs./acre)	Change In Irrigation Water	Change In Net Returns	Change In Net Returns/Change in Irri. Water
1	2	3	4	5	6	7 = 6/5
Small	Model 1	28500	26804.1	-	-	-
	Model 2	28500	26804.1	-	-	-
	Model 3	28500	26804.1	-	-	-
	Model 4	19668	22866.1	8832	3937.98	0.45
	Model 5	11267	14820.2	8401	8045.95	0.96
	Model 6	5633	7410.08	5634	7410.09	1.31
Medium	Model 1	48534	69264.4	-	-	-
	Model 2	43935	65302.3	4599	3962.05	0.86
	Model 3	37096	56337.3	6839	8965.01	1.31
	Model 4	23202	37067.6	13894	19269.8	1.39
	Model 5	11267	18533.8	11935	18533.7	1.55
	Model 6	5633	9266.92	5634	9266.91	1.64
Large	Model 1	54657	74161.6	-	-	-
	Model 2	49617	69366.9	5040	4794.68	0.95
	Model 3	39538	59777.6	10079	9589.34	0.95
	Model 4	22533	41573.6	17005	18204	1.07
	Model 5	11267	20786.8	11266	20786.8	1.84
	Model 6	5633	10393.4	5634	10393.4	1.84

willingness to pay fell down as increasing quantities were utilized. The returns for each farm reduced with decreasing amount of water. The change in net returns with the change in water amount gave the marginal value of water. The value of water, derived from analysis of supply levels were lower than the actual levels supplied during one year, ranged from Rs. 0.45 to Rs. 1.31 in small farm, Rs. 0.86 to Rs. 1.64 in medium farm and Rs. 0.95 to 1.84 in large farm, respectively. The differences in marginal value of water among the three farm sizes are shown in Table III, which were attributed to the effects of land size on the value of water. As the farm size increased, the marginal value of water also increased.

## CONCLUSIONS AND POLICY IMPLICATIONS

The cropping intensity and cropping pattern results showed that cropping intensity reduced due to shortage of water supply. So, the water was the most limiting factor in achieving the higher cropping intensity.

The results also indicated a major shift of cropping pattern from cotton, sorghum and Rabi fodder to wheat, rice, maize and sugarcane. This change occurred, because the wheat, rice, sugar cane and maize required less water during certain months in which water supplies were limited.

In Linear Programming Method, the monthly shadow prices for water showed the relative scarcity of water during the months of Jan., Feb., May, Oct., and Nov. When one month water supply was changed, the shadow price of that month either remained constant or increased as supply of water decreased and vice versa. So, the marginal values of water derived by LP model was in accordance to the definition of the short run MVP (marginal value product).

It is evident from the results of linear programming that the net return from each farm was decreasing with decreasing water supply levels. Sensitivity analysis of the water supply demonstrated diminishing marginal value product and was consistent with the economic theory. Based on the results, following suggestions are advanced for policy makers and ultimately for improving irrigation system:

- Allocation of irrigation water on the basis of the economic value of water.
- Introduction of Public and private initiative partnership on the pattern of PIDA for management of irrigation system at the secondary and tertiary levels.
- Distribution/reallocation of water across provinces and inter-canal command area based on economic value of water rationalizing the use of available water.
- The cropping pattern must be on the basis of consumptive use of water in consultation with the provincial agriculture department.
- The present flat rate policy is neutral; therefore water charges could be levied on the basis of economic value of water.

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